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# Elastic analysis of infilled frames using substructures

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## ABSTRACT

The behaviour of reinforced concrete and steel frames subjected to horizontal loading may be strongly influenced by the presence of masonry infill. In the paper, a mathematical model based on the finite element idealisation is proposed for the linear analysis. Each infill is treated as a substructure and all degrees of freedom corresponding to the infill, with the exception of those at the contact with the frame, are eliminated using the static condensation procedure. The lengths of contact correspond to the condition where separation between frame and infill occured in the infill frame. The procedure is computationally effective for the linear static and free-vibration analysis and preserves the versatility of a finite element approach. It can be easily applied in the practice by using a computer program which includes substructuring option, e.g. SAP84.

## INTRODUCTION

In many countries, steel and/or reinforced concrete structures are filled by brick or concrete block masonry. A continuous contact is usually provided on all sides between frame and infill wall. The experience both from real earthquakes and experiments has proved that the behaviour of frames subjected to horizontal loading may be strongly influenced by infill. Consequently, for a realistic simulation of the actual behaviour infill should be included in mathematical models.

Ordinary building structures subjected to strong ground motion will deform into inelastic range. The inelastic dynamic analysis, however, is too demanding for practical design procedures. The design of earthquake resistant structures is based on linear methods which can, in many cases, not only closely simulate the structural behaviour under minor to moderate loading, but also be used for an approximate simulation of the nonlinear behaviour.

Two types of mathematical models have been widely applied for the linear analysis of infilled frames. An equivalent diagonal strut represents the simplest model which can be used to simulate overall effects of infill after partial separation of frame and infill. The basic idea of the model was proposed by Polyakov (1957), and further developed by other researchers, (e.g. Stafford Smith 1966, Stafford Smith and Carter 1969, Mainstone 1971). Another possibility is to model infill walls with finite elements. Such a model is much more versatile, provides results also on local levels and can be easily used for an infill wall with arbitrary openings and for parapet walls. A finite element model was first used by Mallick and Severn (1967) and later applied in different variants by other researchers (e.g. Moss and Carr 1971, Mallick and Garg 1971, Riddington and Stafford Smith 1977, King and Pandey 1978).

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Some attempts have been made to develop different models, e.g. a multidiagonal model (Thiruvengadam 1985), a model with an eccentric diagonal (Žarnić 1990), a model with two diagonals (Schmidt 1990), and a panel model (Axley and Bertero 1979). In the latter approach the infill was first modeled by finite elements. Then, all degrees of freedom, with the exception of the displacements and rotations of the corner nodes, were eliminated by the static condensation procedure. By using different assumptions, four different stiffness matrices, representing different types of infill, were developed in closed forms. The procedure is computationally effective but restricted to the predefined types of infill frames.

The procedure described in this paper represents a combination of computing efficiency (not much lower as in the case of a diagonal strut model) and versatility of a finite element approach. Infill walls are modelled with finite elements and treated as substructures which are connected to frame only in a few points. Very large building structures can be efficiently analysed by using a general purpose finite element program with substructuring option (e.g. SAP84 ). In the paper the mathematical model and the method of analysis are described and some numerical examples are presented.

## MATHEMATICAL MODEL

The mathematical model and the method of analysis described in this paper can be in principle applied for any type of infill frames at any load level. We will study, however, only infill frames where no special connectors are provided to ensure contact between frame and infill. In such structures, separation between frame and infill occurs due to differences between flexural deformations of the frame and shear deformations of the infill panel (Fig. 1) which produce a tension failure of the connection. This separation may occur at a load level of approximately half of the ultimate capacity of the infill frame. If the linear analysis is used, mathematical models typically simulate the situation after bond separation (e.g. model with equivalent diagonal strut). This situation can be easily reproduced in a finite element analysis, if no tension strength is assumed at the connection between frame and infill.

In the mathematical model, applied in our study for static and free-vibration analysis, beam elements with 3 degrees of freedom per node will be used to model the frame and rectangular plane stress elements with 4 nodes and 2 degrees of freedom per node (panel elements) will be used to model the infill. Both types of elements will be rigidly connected in the common nodes (the same displacements will be assumed). No connection between beam and panel elements will be provided in the regions where separation between frame and infill occurs.

The zones and the lengths of contact between frame and infill can be determined by iteration. First, a rigid contact is assumed in all common nodes. Then, the rigid connection between beam and panel elements is removed in all regions where tension occurs at the connection. The procedure rapidly converges towards the final stage. It should be noted, however, that the real length of contact depends not only on structural parameters which are included in the analysis, but also on the variations in the quality of material and of workmanship which cannot be taken into account in the computation. Fortunately, the results of parametric studies have demonstrated that the main results (with the exception of the stresses in the corners of infill) are only scarcely dependent on small variations of the lengths of contact. For these reasons, it is not reasonable to attempt to calculate "exact" lengths of contact. In usual cases, the length of contact $\alpha$ of a column can be determined using the formula proposed by Stafford Smith (1966) (Fig. 1)

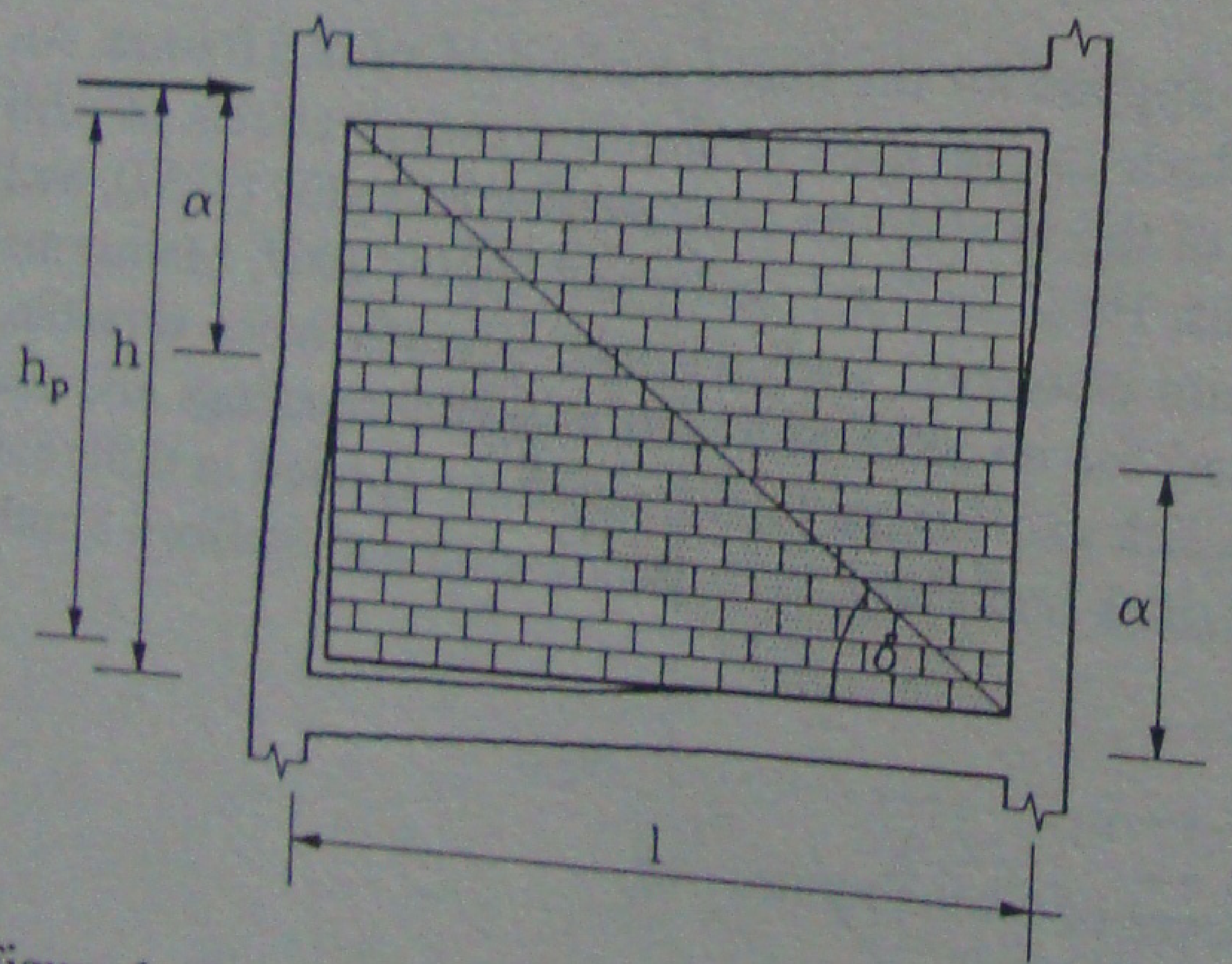


Figure 1. Infilled frame (after separation between frame and infill)

$$\frac{a}{h} = \frac{\pi}{2 \lambda h} \tag{1}$$

where $\lambda h$ is a nondimensional parameter dependent on the ratio of the stiffness of the infill to the stiffness of the frame

$$\lambda h = h \sqrt[4]{\frac{E_p \, t_p \sin 2\delta}{4 E I h_p}} \tag{2}$$

$E_p$ and $E$ are the modulus of elasticity of infill and frame, respectively, $I$ is the moment of inertia of the column, $t_p$ is the thickness of the infill. The meaning of other parameters can be seen in Fig. 1.

The length of contact of a beam is less important and can be either assumed to be approximately equal to one half of the length of the beam or determined according to the formulae analogous to Eqs. 1 and 2. In the case of a partial infill (parapets, infill witht openings) where the lengths of contact are not known in advance, it may be necessary to determine them by iteration.

Some mathematical models, reported in the literature, consider the influence of sliding at the connection between frame and infill. In principle, this influence can be easily included in the presented method. It requires, however, an iterative procedure and has only a small effect on results, with the exception of the stresses in the infill in the vicinity of the contact ((Riddington and Stafford Smith 1977). For these reasons sliding at the contact zones was not included in the mathematical model.

In a finite element analysis the accuracy of results depends on the density of the mesh of finite elements. It has been found that an accuracy appropriate for design purposes can be obtained by using a 8 x 8 mesh in the panel. In many cases even a 4 x 4 mesh may yield acceptable results for displacements.

## METHOD OF ANALYSIS

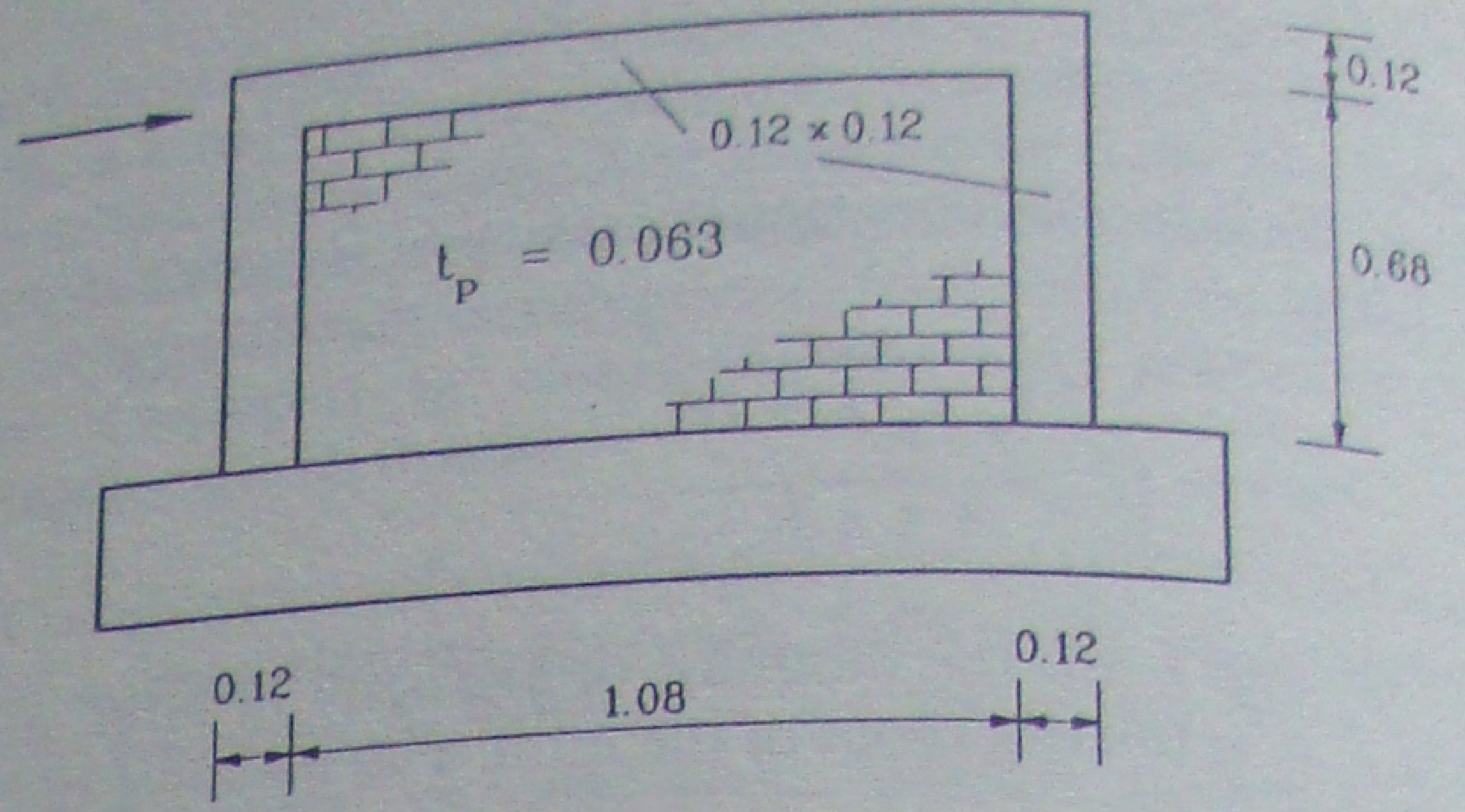
An analysis of a mathematical model of a real multistory building with infilled frames, which may have several thousands degrees of freedom, is computationaly demanding and its results are hard to be looked over. Computational efficiency and survaillance of results will be greatly improved if substructuring technique is used. This technique is ideally suited for the analysis of infilled frames where typically identical infill walls are provided in different stories and different bays.

Each infill is treated as one substructure. The stiffness matrix for each substructure is first formulated by the finite-element approach. Then, all degrees of freedom with the exception of those belonging to the nodes at the contact with the frame are eliminated from the stiffness matrix by the static condensation procedure (e.g. Przemieniecki 1968). Finally, the stiffness matrix of the whole structure is formed. Only degrees of freedom which correspond to the contact points between frame and infill are included. The number of degrees of freedom is larger than in the case of a bare frame or in the case of an infilled frame modeled with equivalent diagonals, but it is an order of magnitude lower than in the case of the classical finite element approach. The same model and the same substructuring procedure can be used for the static and free-vibration analysis.

## COMPARISON WITH EXPERIMENTS

In the Institute for Testing and Research in Materials and Structures in Ljubljana a series of experiments on 1:3 models of infilled reinforced concrete single-story single bay frames has been made (Žarnić 1990). The test structure is shown in Fig. 2. The observed horizontal force - top displacement relation is shown in Fig. 3. Relatively large standard deviation from the mean value can be observed even under laboratory conditions mainly due to the variation in the quality of workmanship. Two different mathematical models were used in the analysis. The first model was intended to reproduce the initial behaviour of the structure under small loading. A rigid contact between frame and brick masonry infill was assumed on all sides. The shear modulus of the infill $G_p$ was determined according to the theory of elasticity, using Poisson's coefficient $\nu = 0.1$. As shown in Fig. 3, the stiffness of the mathematical model corresponds very well to the initial mean stiffness of the test models.

The second model was designed to simulate the state of the structure after the separation of the infill from the frame. The contact between the frame and the infill was provided only in the contact zone determined according to Eq. 1. A lower shear modulus, based on experiments on brick masonry walls subjected to large horizontal loading, was used. The stiffness of the mathematical model corresponds to the stiffness of the test models at approximately 50 % to 60 % of the ultimate loading, as shown in Fig. 3.



CONCRETE: $E = 1.1 \times 10^7$
$G = 4.4 \times 10^6$

INFILL: $E_p = 3.857 \times 10^6$
$G_p = 1.753 \times 10^6$ (model 1)
$G_p = 0.505 \times 10^6$ (model 2)

Figure 2. Test structure (units are meters and kilonewtons)

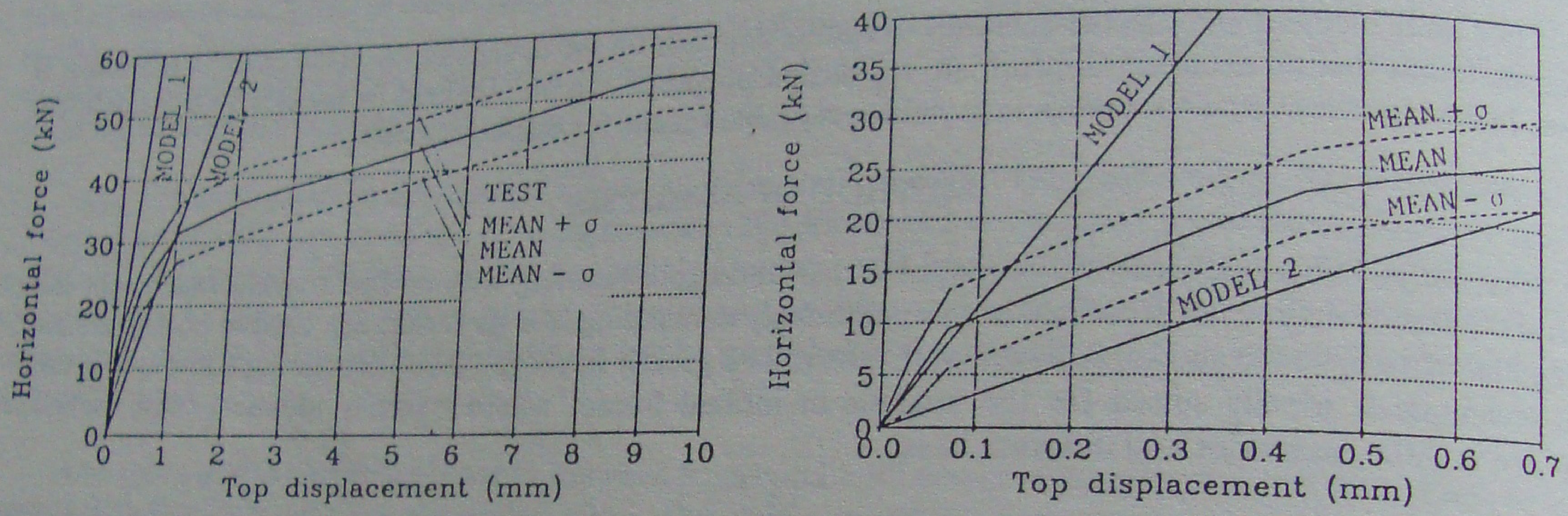


Figure 3. Horizontal force - top displacement relationships obtained in tests and analyses. The figure on the right side represents a detail of the figure on the left side

## PARAMETRIC STUDY OF AN INFILLED FRAME WITH OPENINGS

The advantage of the proposed procedure in comparison to the classical finite element approach can be fully appreciated if analysing multi-story multi-bay infilled frames. Due to the space limitation, however, in this paper only some results of a parametric study of a simple single-story single-bay infilled frame with different types of openings are presented as an example of the application of the analysis procedure. The program SAP84, which includes the substructuring option, was used for the analysis. SAP84 has been originally developed at Peking University. It has been partially modified and extended in Ljubljana (Yuan and Peruš, 1988). The U.S. version of the program is called MICSAP (Yuan and Chang 1990).

Two horizontal forces (50 kN each) act in the upper two corners of the structure. The structural characteristics of the infilled frame are as follows (units are meters and kilonewtons)

$h = 4,\ l = 4,$ $\lambda h = 6.2$
Columns and beams 0.40/0.40
Panel $t_p = 0.20$
$E = 3.2 \times 10^7$ $G = 1.37 \times 10^7$
$E_p = 3.2 \times 10^6$ $G_p = 1.37 \times 10^6$

In addition to the bare frame (Model B), five different infilled frames have been studied (Fig. 4). The length of contact in the model F was determined according to Eq. 1. In the case of other models the lenghts of contact were defined using an iterative procedure.

Some typical results are shown in Figs. 5 to 7. Comparison of displacements and internal forces in the left column of the investigated models are shown in Fig. 5. The stiffness of infilled framed is up to ten times greater than the stiffness of the bare frame. Maximum tensile axial force in the left column of infilled frames is larger than in the column of the bare frame. Maximum shear force in the left column in the case of parapet infill is larger than in the column of the bare frame. Maximum values of bending moments in the column are observed at the top of parapets. It should be noted that the comparison in Fig. 5 was made at the same horizontal loading. In reality, however, usually much larger forces will be attracted to infilled frames due to their much larger stiffness. Maximum compresive stresses in the infill of four models are shown in Fig. 6 and the principal stresses are shown in Fig. 7 (tensile stresses are marked with arrows). It should be noted that the details of the stresses in the corners of the completely filled frame are strongly influenced by the parameter $\lambda h$ and might be numerically sensitive. High stress concentrations can be observed near to the corner of the opening in Model O and at the left upper corner of the parapet walls. A very distinctive diagonal path of compressive stresses can be observed in the case of the completely filled frame (Model F). Diagonals can be observed also in the case of parapet walls. However, they do not reach the bottom corner on the right side. A distorted diagonal path can be seen in the infill with an opening (Model O).

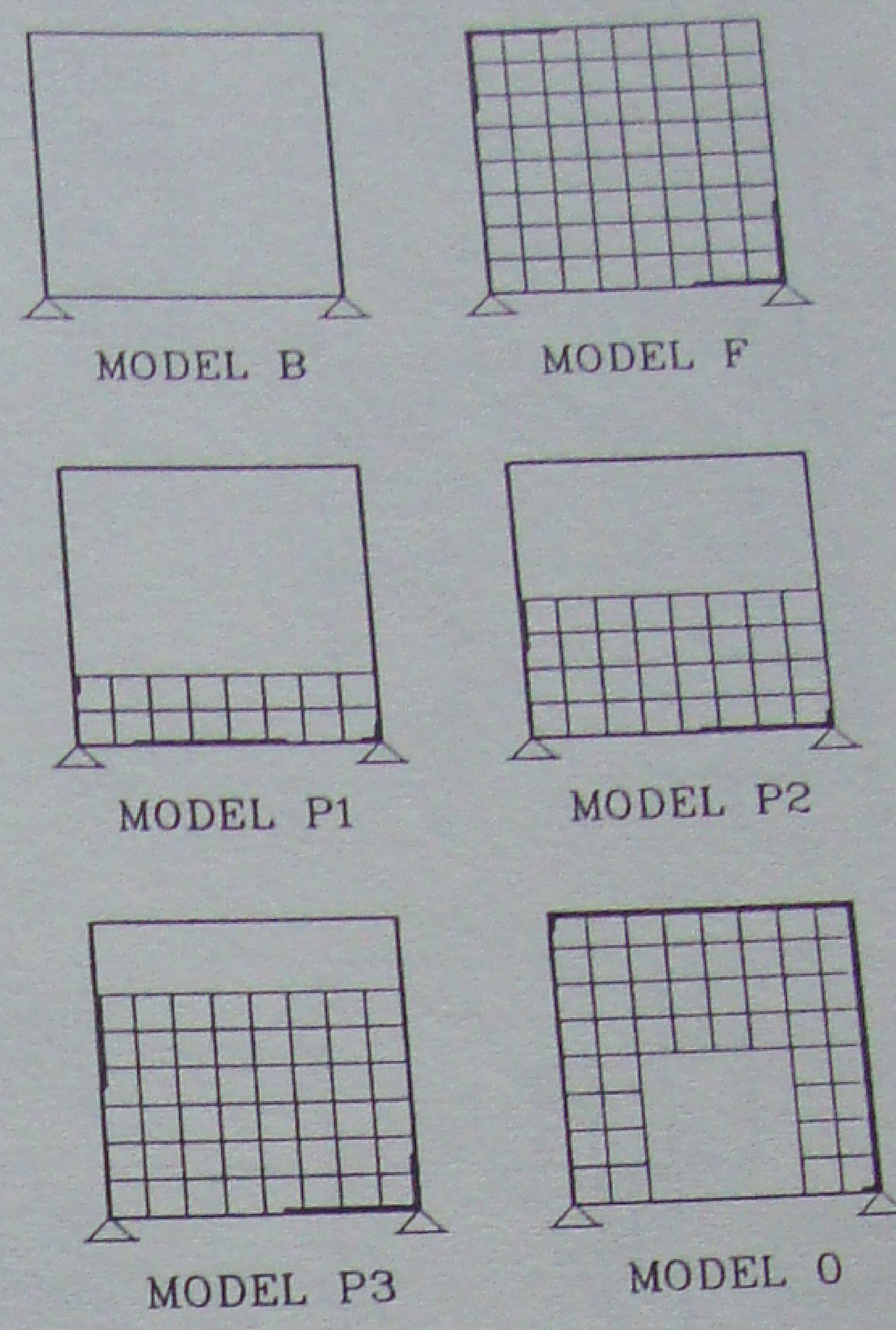


Figure 4. Models analysed in the parametric study. The mesh of finite elements in the infill and the length of contacts are shown in each model

## CONCLUSIONS

The main findings of the research reported in (Reflak 1990), and partly summarized in this paper, are as follows.

Masonry infill can drastically alter the structural response of frames and of whole structures. Thus, for a realistic simulation of the behaviour of infilled frames subjected to horizontal loading infill should be included in the mathematical model. A model with an equivalent diagonal strut is the most usual model for the elastic analysis. It can predict the stiffness of a completely infilled frame with resonable accuracy. It may fail, however, in the case of the infill with openings. It also usually does not provide correct results for shear forces and bending moments in the columns of the frame. (Shear failure of columns is a frequent failure mode for masonry infilled frames.)

A finite element model can be easily applied for any type of infill. Partial separation of frame and infill is usually assumed in the case of an elastic analysis. The lenghts of contact of infill with the surrounding frame can be determined by Eqs. 1 and 2, or, in the case of an infill with openings and unknown behaviour, by an iteration procedure. Provided that an appropriate mesh is used, a finite element model yields correct results for

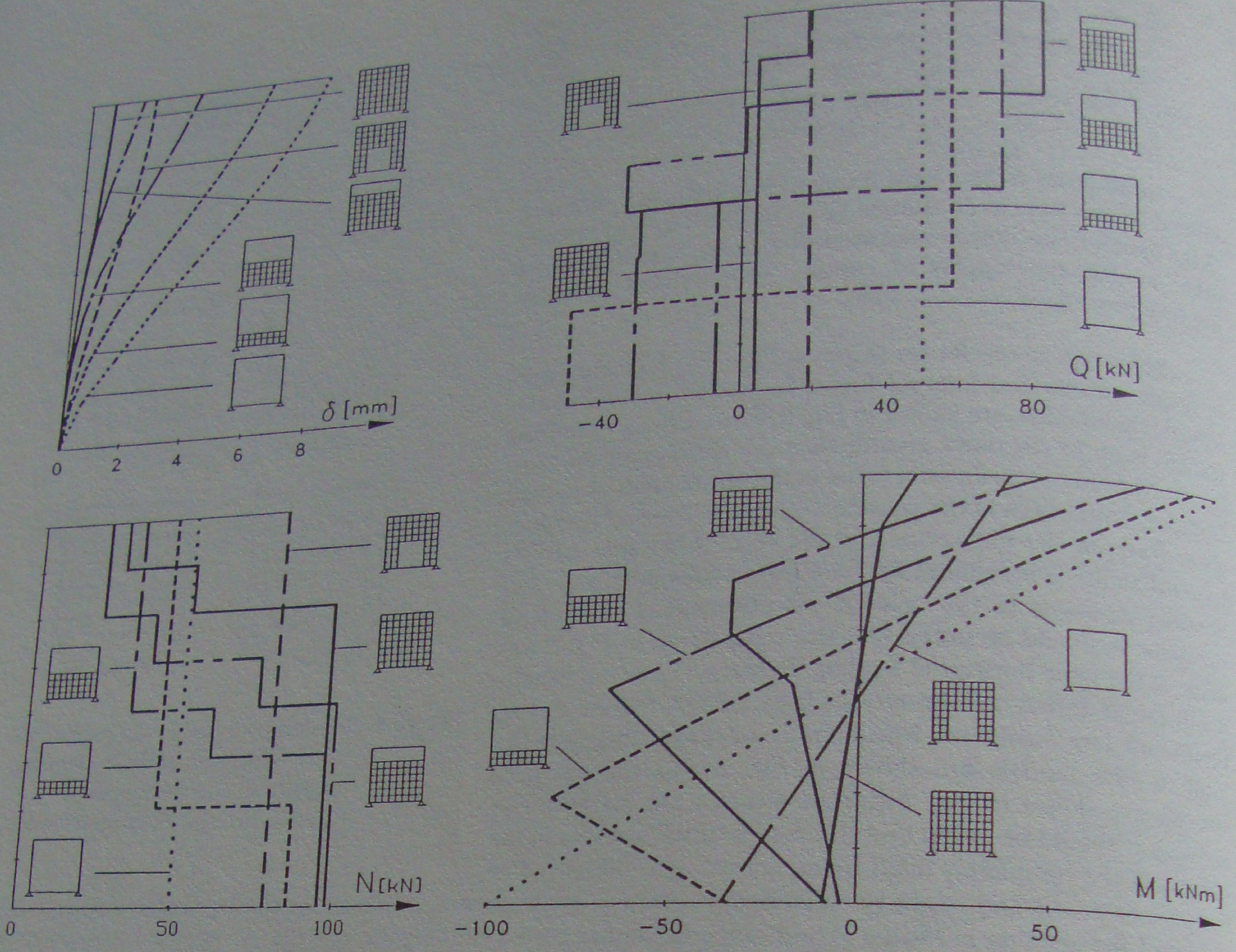


Figure 5. Displacements $\delta$ and internal forces (shear force $Q$, axial force $N$ and bending moment $M$) in the left column

displacements, internal forces in frame and stresses in infill, except of the local stresses in the vicinity of the contact between frame and infill, especially in the corners of the infill. These stresses are extremely dependent on many details of the mathematical model. Some of the details (e.g. the exact length of contact between frame and infill and the detailed behaviour of the contact) depend a lot on the quality of the workmanship and cannot be accurately predicted even in the case of laboratory conditions. For these reasons, any calculated local stress in the vicinity of the contact can be only a very rough approximation. The shear modulus of infill $G_p$ may have a relatively important influence on the behaviour of infilled frames. At higher loads it does not follow the theory of elasticity. It should be based on the experiments on masonry walls, if available.

A finite element model may have in case of a multistory multibay infilled frame several thousands degrees of freedom and be prohibitively restrictive for the analysis of an ordinary building in a design office. However, when using the substructuring technique, treating each infill as one substructure, and eliminating all degrees of freedom cor-responding to the infill but those in the contact joints with the frame, the feasibility of the analysis will be largely improved.

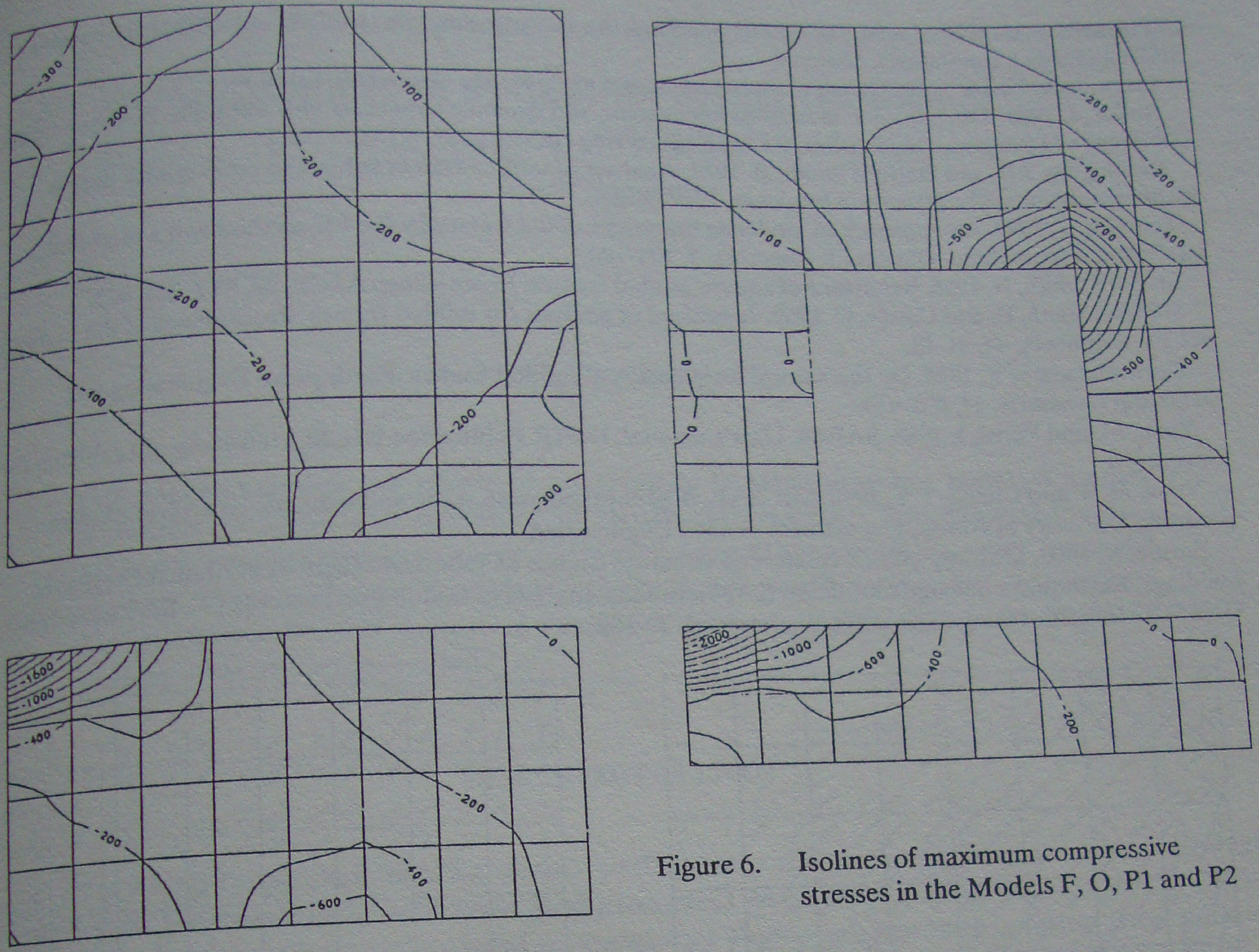


Figure 6. Isolines of maximum compressive stresses in the Models F, O, P1 and P2

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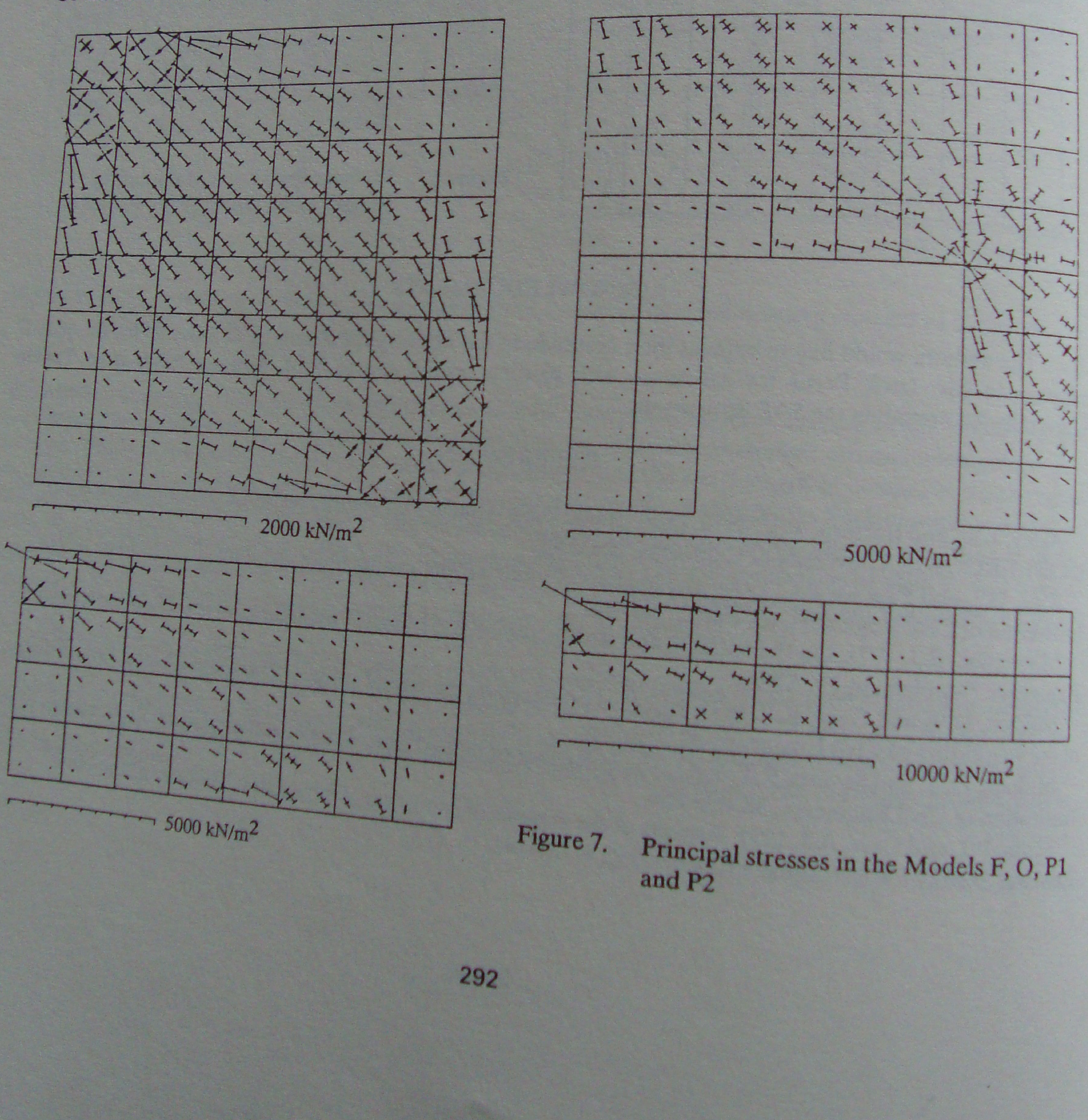

Figure 7. Principal stresses in the Models F, O, P1 and P2